

Work hardening, also known as **strain hardening**, is the strengthening of a metal by plastic deformation.

This strengthening occurs because of dislocation movements within the crystal structure of the material.

Any material with a reasonably high melting point such as metals and alloys can be strengthened in this fashion^{[Melting point =157°C}

Alloys not amenable to heat treatment, including low-carbon steel, are often work-hardened. Some materials cannot be work-hardened at normal ambient temperatures, such as indium,^[citation needed] however others can only be strengthened via work hardening, such as pure copper and aluminum.

An example of undesirable work hardening is during machining when early passes of a cutter inadvertently work-harden the workpiece surface, causing damage to the cutter during the later passes. An example of desirable work hardening is that which occurs in metalworking processes that intentionally induce plastic deformation to exact a shape change. These processes are known as **cold working** or **cold forming** processes.

In metallic crystals, irreversible deformation is usually carried out on a microscopic scale by defects called dislocations, which are created by fluctuations in local stress fields within the material culminating in a lattice rearrangement as the dislocations propagate through the lattice. At normal temperatures the dislocations are not annihilated by annealing.

Instead, the dislocations accumulate, interact with one another, and serve as pinning points or obstacles that significantly impede their motion. This leads to an increase in the yield strength of the material and a subsequent decrease in ductility.

The effects of cold working may be reversed by annealing the material at high temperatures where recovery and recrystallization reduce the dislocation density.

Recovery is a process by which deformed grains can reduce their stored energy by the removal or rearrangement of defects in their crystal structure. These defects, primarily dislocations, are introduced by plastic deformation of the material and act to increase the yield strength of a material. Since recovery reduces the dislocation density the process is normally accompanied by a reduction in a materials strength and a simultaneous increase in the ductility. As a result recovery may be considered beneficial or detrimental depending on the circumstances.

Recovery is related to the similar process of recrystallisation and grain growth. Recovery competes with recrystallisation, as both are driven by the stored energy, but is also thought to be a necessary prerequisite for the nucleation of recrystallised grains.

Recrystallization is a process by which deformed grains are replaced by a new set of undeformed grains that nucleate and grow until the original grains have been entirely consumed. Recrystallization is usually accompanied by a reduction in the strength and hardness of a material and a simultaneous increase in the ductility.

Nucleation

The recrystallised grains do not nucleate in the classical fashion but rather grow from pre-existing sub-grains and cells. The 'incubation time' is then a period of recovery where sub-grains with low-angle boundaries ($<1-2^\circ$) begin to accumulate dislocations and become increasingly misoriented with respect to their neighbours. The increase in misorientation increases the mobility of the boundary and so the rate of growth of the sub-grain increases. If one sub-grain in a local area happens to have an advantage over its neighbours (such as locally high dislocation densities, a greater size or favourable orientation) then this sub-grain will be able to grow more rapidly than its competitors. As it grows its boundary becomes increasingly misoriented with respect to the surrounding material until it can be recognised as an entirely new strain-free grain.

Grain growth refers to the increase in size of grains (crystallites) in a material at high temperature. This occurs when recovery and recrystallisation are complete and further reduction in the internal energy can only be achieved by reducing the total area of grain boundary. The term is

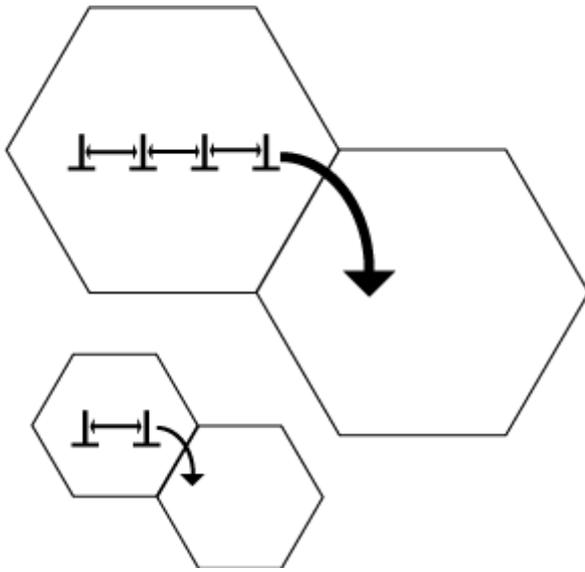
commonly used in metallurgy but is also used in reference to ceramics and minerals.

Cold working generally results in a higher yield strength as a result of the increased number of dislocations and the Hall-Petch effect of the sub-grains, and a decrease in ductility.

Grain-boundary strengthening (or Hall-Petch strengthening) is a method of strengthening materials by changing their average crystallite (grain) size. It is based on the observation that grain boundaries impede dislocation movement and that the number of dislocations within a grain

have an effect on how easily dislocations can traverse grain boundaries and travel from grain to grain. So, by changing grain size one can influence dislocation movement and yield strength. For example, heat treatment after plastic deformation

and changing the rate of solidification are ways to alter grain size.



The smaller the grain size, the smaller the repulsion stress felt by a grain boundary dislocation and the higher the applied stress needed to propagate dislocations through the material.

The relation between yield stress and grain size is described mathematically by the Hall-Petch equation:

$$\sigma_y = \sigma_0 + \frac{k_y}{\sqrt{d}}$$

where σ_y is the yield stress, σ_0 is a materials constant for the starting stress for dislocation movement (or the resistance of the lattice to dislocation motion), k_y is the strengthening coefficient (a constant unique to each material), and d is the average grain diameter.

Theoretically, a material could be made infinitely strong if the grains are made infinitely small. This is, unfortunately, impossible because the lower limit of grain size is a single [unit cell](#) of the material. Even then, if the grains of a material are the size of a single unit cell, then the material is in fact amorphous, not crystalline, since there is no long range order, and dislocations can not be defined in an amorphous material. It has been observed experimentally that the microstructure

with the highest yield strength is a grain size of about 10 nanometers, because grains smaller than this undergo another yielding mechanism, grain boundary sliding. Producing engineering materials with this ideal grain size is difficult because only thin films can be reliably produced with grains of this size.

This is a schematic roughly illustrating the concept of dislocation pile up and how it effects the strength of the material. A material with larger grain size is able to have more dislocation to pile up leading to a bigger driving force for dislocations to move from one grain to another. Thus you will have to apply less force to move a dislocation from a larger than from a smaller grain, leading materials with smaller grains to exhibit higher yield stress.

| There are two common mathematical descriptions of the work hardening phenomenon. Hollomon's equation is a power law relationship between the stress and the amount of plastic strain: | Hall-Petch constants ¹ | | |
|---|-----------------------------------|------------------|-----------------------------|
| | Material | σ_0 [MPa] | k [MPa m ^{1/2}] |
| | Copper | 25 | 0.11 |
| | Titanium | 80 | 0.40 |
| | | | 0.74 |

$$\sigma = K \epsilon_p^n \quad \text{mild steel } 70$$

where σ is the stress, K is the strength index, ϵ_p is the plastic strain and n is the [strain hardening index](#). Ludwik's equation is similar but includes the yield stress:

$$\sigma = \sigma_y + K\epsilon_p^n$$

If a material has been subjected to prior deformation (at low temperature) then the yield stress will be increased by a factor depending on the amount of prior plastic strain ϵ_0 :

$$\sigma = \sigma_y + K(\epsilon_0 + \epsilon_p)^n$$

The constant K is structure dependent and is influenced by processing while n is a material property normally lying in the range 0.2–0.5. The strain hardening index can be described by:

$$n = \frac{d \log(\sigma)}{d \log(\epsilon)} = \frac{\epsilon}{\sigma} \frac{d\sigma}{d\epsilon}$$

This equation can be evaluated from the slope of a $\log(\sigma)$ - $\log(\epsilon)$ plot. Rearranging allows a determination of the rate of strain hardening at a given stress and strain:

$$\frac{d\sigma}{d\epsilon} = n \frac{\sigma}{\epsilon}$$

The engineering stress-strain curve does not give a true indication of the deformation characteristics of a metal because it is based entirely on the original dimensions of the specimen, and these dimensions change continuously during the test. Also, ductile metal which is pulled in tension becomes unstable and necks down during the course of the test. Because the cross-sectional area of the specimen is decreasing rapidly at this stage in the test, the load required continuing deformation falls off. The average stress based on original area likewise decreases, and this produces the fall-off in the stress-strain curve beyond the point of maximum load. Actually, the metal

continues to strain-harden all the way up to fracture, so that the stress required to produce further deformation should also increase. If the true stress, based on the actual cross-sectional area of the specimen, is used, it is found that the stress-strain curve increases continuously up to fracture. If the strain measurement is also based on instantaneous measurements, the curve, which is obtained, is known as a **true-stress-true-strain curve**. This is also known as a flow curve since it represents the basic plastic-flow characteristics of the material. Any point on the flow curve can be considered the yield stress for a metal strained in tension by the amount shown on the curve. Thus, if the load is removed at this point and then reapplied, the material will behave elastically throughout the entire range of reloading.

The true stress σ is expressed in terms of engineering stress s by

$$\sigma = \frac{P}{A_0} \cdot (e + 1) = s \cdot (e + 1) \quad (1)$$

The derivation of Eq. (1) assumes both constancy of volume and a homogenous distribution of strain along the gage length of the tension specimen. Thus, Eq. (1) should only be used until the onset of necking. Beyond maximum load the true stress should be determined from actual measurements of load and cross-sectional area.

$$\sigma = \frac{P}{A} \quad (2)$$

The true strain ϵ may be determined from the engineering or conventional strain e by

$$\epsilon = \ln (e + 1) \quad (3)$$

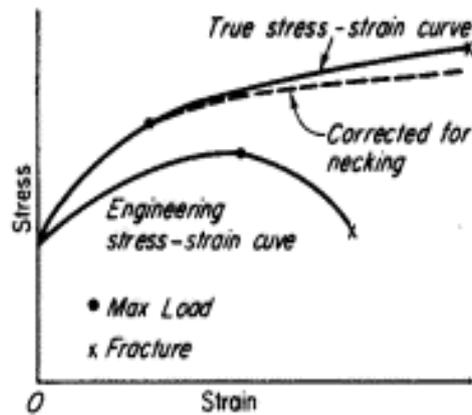


Figure 1. Comparison of engineering and true stress-strain curves

This equation is applicable only to the onset of necking for the reasons discussed above. Beyond maximum load the true strain should be based on actual area or diameter measurements.

$$\varepsilon = \ln \frac{A_0}{A} = \ln \frac{(\pi/4)D_0^2}{(\pi/4)D^2} = 2 \ln \frac{D_0}{D} \quad (4)$$

Figure 1 compares the true-stress-true-strain curve with its corresponding engineering stress-strain curve. Note that because of the relatively large plastic strains, the elastic region has been compressed into the y-axis. In agreement with Eqs. (1) and (3), the true-stress-true-strain curve is always to the left of the engineering curve until the maximum load is reached. However, beyond maximum load the high-localized strains in the necked region that are used in Eq. (4) far exceed the engineering strain calculated from Eq. (1). Frequently the flow curve is linear from maximum load to fracture, while in other cases its slope continuously decreases up to fracture. The formation of a necked region or mild notch introduces triaxial stresses, which make it difficult to determine accurately the longitudinal tensile stress on out to fracture.

The following parameters usually are determined from the true-

stress-true-strain curve.

True Stress at Maximum Load

The true stress at maximum load corresponds to the true tensile strength. For most materials necking begins at maximum load at a value of strain where the true stress equals the slope of the flow curve. Let σ_u and ϵ_u denote the true stress and true strain at maximum load when the cross-sectional area of the specimen is A_u . The ultimate tensile strength is given by

$$\begin{aligned} s_u &= \frac{P_{\max}}{A_0} \\ \sigma_u &= \frac{P_{\max}}{A_u} \\ \epsilon_u &= \ln \frac{A_0}{A_u} \end{aligned}$$

Eliminating P_{\max} yields

$$\sigma_u = s_u \frac{A_0}{A_u} \qquad \sigma_u = s_u e^{\epsilon_u} \qquad (5)$$

True Fracture Stress

The true fracture stress is the load at fracture divided by the cross-sectional area at fracture. This stress should be corrected for the, triaxial state of stress existing in the tensile specimen at fracture. Since the data required for this correction are often not available, true-fracture-stress values are frequently in error.

True Fracture Strain

The true fracture strain ϵ_f is the true strain based on the original area A_0 and the area after fracture A_f

$$\epsilon_f = \ln \frac{A_0}{A_f} \qquad (6)$$

This parameter represents the maximum true strain that the material can withstand before fracture and is analogous to the total strain to fracture of the engineering stress-strain curve. Since Eq. (3) is not valid beyond the onset of necking, it is not

possible to calculate ε_f from measured values of ε_f . However, for cylindrical tensile specimens the reduction of area q is related to the true fracture strain by the relationship

$$\varepsilon_f = \ln \frac{1}{1 - q} \quad (7)$$

True Uniform Strain

The true uniform strain ε_u is the true strain based only on the strain up to maximum load. It may be calculated from either the specimen cross-sectional area A_u or the gage length L_u at maximum load. Equation (3) may be used to convert conventional uniform strain to true uniform strain. The uniform strain is often useful in estimating the formability of metals from the results of a tension test.

$$\varepsilon_u = \ln \frac{A_0}{A_u} \quad (8)$$

True Local Necking Strain

The local necking strain ε_n is the strain required to deform the specimen from maximum load to fracture.

$$\varepsilon_n = \ln \frac{A_u}{A_f} \quad (9)$$

The flow curve of many metals in the region of uniform plastic deformation can be expressed by the simple power curve relation

$$\sigma = K \cdot \varepsilon^n \quad (10)$$

where n is the **strain-hardening exponent** and K is the **strength coefficient**. A log-log plot of true stress and true strain up to maximum load will result in a straight-line if Eq. (10) is satisfied by the data (Fig. 1). The linear slope of this line is n and K is the true stress at $\varepsilon = 1.0$ (corresponds to $q = 0.63$). The strain-hardening exponent may have values from n

= 0 (perfectly plastic solid) to $n = 1$ (elastic solid) (see Fig. 2). For most metals n has values between 0.10 and 0.50 (see Table 1.).

It is important to note that the **rate of strain hardening** $d\sigma / d\epsilon$, is not identical with the strain-hardening exponent.

From the definition of n

$$n = \frac{d(\log \sigma)}{d(\log \epsilon)} = \frac{d(\ln \sigma)}{d(\ln \epsilon)} = \frac{\epsilon}{\sigma} \frac{d\sigma}{d\epsilon}$$

or

$$\frac{d\sigma}{d\epsilon} = n \frac{\sigma}{\epsilon} \tag{11}$$

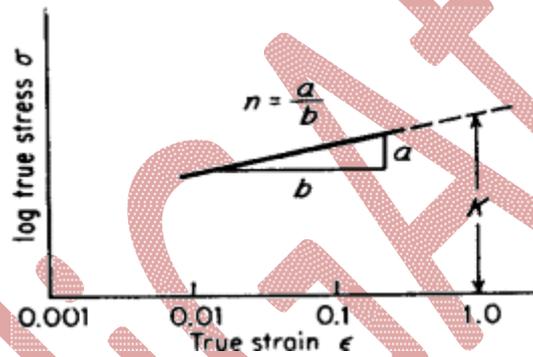


Figure 2. Log/log plot of true stress-strain curve

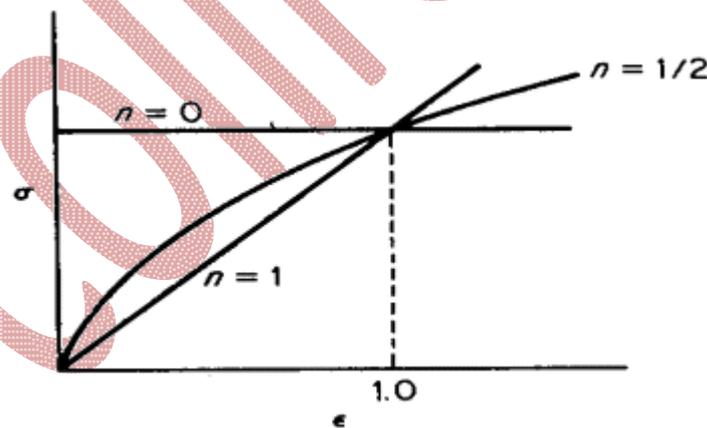


Figure 3. Various forms of power curve $\sigma = K^* \epsilon^n$

Table 1. Values for n and K for metals at room temperature

| Metal | Condition | n | K , psi |
|---------------|-----------|------|-----------|
| 0,05% C steel | Annealed | 0,26 | 77000 |

| | | | |
|-----------------------|---------------------------------|------|--------|
| SAE 4340 steel | Annealed | 0,15 | 93000 |
| 0,60% C steel | Quenched and tempered 1000°F | 0,10 | 228000 |
| 0,60% C steel | Quenched and tempered 1300°F | 0,19 | 178000 |
| Copper | Annealed | 0,54 | 46400 |
| 70/30 brass | Annealed | 0,49 | 130000 |

There is nothing basic about Eq. (10) and deviations from this relationship frequently are observed, often at low strains (10^{-3}) or high strains ($\epsilon \approx 1,0$). One common type of deviation is for a log-log plot of Eq. (10) to result in two straight lines with different slopes. Sometimes data which do not plot according to Eq. (10) will yield a straight line according to the relationship

$$\sigma = K(\epsilon_0 + \epsilon)^n \quad (12)$$

Datsko has shown how ϵ_0 , can be considered to be the amount of strain hardening that the material received prior to the tension test.

Another common variation on Eq. (10) is the Ludwig equation

$$\sigma = \sigma_0 + K \cdot \epsilon^n \quad (13)$$

where σ_0 is the yield stress and K and n are the same constants as in Eq. (10). This equation may be more satisfying than Eq. (10) since the latter implies that at zero true strain the stress is zero. Morrison has shown that σ_0 can be obtained from the intercept of the strain-hardening point of the stress-strain curve and the elastic modulus line by

$$\sigma_0 = \left(\frac{K}{E^n} \right)^{\frac{1}{1-n}}$$

The true-stress-true-strain curve of metals such as austenitic

stainless steel, which deviate markedly from Eq. (10) at low strains, can be expressed by

$$\sigma = K \cdot \varepsilon^n + e^{K_1} \cdot e^{n_2 \varepsilon}$$

where e^{K_1} is approximately equal to the proportional limit and n_2 is the slope of the deviation of stress from Eq. (10) plotted against ε . Still other expressions for the flow curve have been discussed in the literature.

The true strain term in Eqs.(10) to (13) properly should be the **plastic strain**

$$\varepsilon_p = \varepsilon_{\text{total}} - \varepsilon_E = \varepsilon_{\text{total}} - \sigma/E$$

A **Lüder band** is a localized band of [plastic deformation](#) that can occur on some materials before [fracture](#).

These bands can arise from residual [stresses](#) due to [welding](#).

Lüder bands often are a result of [strain aging](#) by discontinuous [yielding](#) and can be commercially important for producing wrinkled lined finishes.

It happens on low carbon steels and some Al-Mg Alloys.

Steel generally exhibits a very linear stress–strain relationship up to a well defined yield point (figure 1). The linear portion of the curve is the elastic region and the slope is the modulus

of elasticity or Young's Modulus. After the yield point, the curve typically decreases slightly because of dislocations escaping from Cottrell atmospheres. As deformation continues, the stress increases on account of strain hardening until it reaches the ultimate strength. Until this point, the cross-sectional area decreases uniformly because of Poisson contractions. The actual rupture point is in the same vertical line as the visual rupture point.

However, beyond this point a *neck* forms where the local cross-sectional area decreases more quickly than the rest of the sample resulting in an increase in the true stress. On an engineering stress–strain curve this is seen as a decrease in the stress. Conversely, if the curve is plotted in terms of *true stress* and *true strain* the stress will continue to rise until failure. Eventually the neck becomes unstable and the specimen ruptures (fractures).

Less ductile materials such as aluminum and medium to high carbon steels do not have a well-defined yield point. For these materials the yield strength is typically determined by the "offset yield method", by which a line is drawn parallel to the linear elastic portion of the curve and intersecting the abscissa at some arbitrary value (most commonly 0.2%). The intersection of this line and the stress–strain curve is reported as the yield point.